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Mass thickness measurements for dual-component samples utilizing equivalent energy of X-rays*

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In this paper, equivalent energy method is introduced for measuring mass thickness of dual-component samples using dual-energy X-rays. Approximately, the method adopts equivalent mass attenuation coefficients of the two components in mass thickness measurements for dual-component samples, in a certain range of thicknesses. Feasibility of the method is proven by numerical calculations and Monte Carlo simulations (EGSnrc package). The results of absorption experiments using an X-ray machine at tube voltages of 30 and 45 kV, the relative errors are less than 5% between the nominal and detected values. Also, optical low energy is discussed at given high voltages.

Keywords: X-rays, Optimal dual-energy, Dual-sample, Mass thickness

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I. INTRODUCTION

X-ray sources are widely used in non-destructive test, in which the beam-hardening artifact that limits the ability of X-ray quantitative analysis needs to be eliminated [1, 2]. In medical applications, optimal energies for dual-energy computed tomography and optimal tube voltage selection for dual-energy imaging of the chest were studied, and problems of optimal dual-energy CT in different parts of medical fields were discussed [3, 4]. In this paper, mass thickness measurements for dual-component samples is investigated, and dual-component samples of aluminum and plexiglass (PMMA, poly(methyl methacrylate)) are used as phantom materials in bone density measurements.

In our previous studies, the equivalent energy method was introduced in the measurement for a single sample [5], and a log-linear rate of absorption was found for a range of material thicknesses when an extra filter was used, proved by numerical calculations and measurements. In this study, this method was applied to mass thickness measurements of dual-component samples (Al and PMMA), by numerical calculations and absorption experiments.

II. MATERIAL AND METHODS

A. Equivalent energy method

Electron beam-induced X-rays consist of continuous and characteristic X-rays. In the equivalent energy method, a suitable filter is used to absorb the characteristic photons and to pre-harden X-ray spectra at a special voltage. The hardened spectra have a peaked energy distribution and the beam-hardening effect can be ignored for a certain range of sample thicknesses.

B. Numerical calculations

Considering the effects of photoemission, Compton scattering, Rayleigh scattering and electron pair effect ($E > 1.02$ MeV), the photon absorption in materials can be well computed. And the results of mass thickness measurements can be calculated through numerical calculations at the base of Kramer's continuous spectra formula and the detection principles.

For current-mode detection, the current signals from a dual-materials in mass thicknesses of M_1 and M_2 are proportional to photon energy and number of photons deposited in the detector. For dual-component samples, the Eq. (1) can be obtained,

$$\begin{cases} i(M_1, M_2) = k\displaystyle\int_0^{E_0} qE(E_0 - E)e^{-\mu_1(E)M_1-\mu_2(E)M_2}\mathrm{d}E, \\ i'(M_1, M_2) = k\displaystyle\int_0^{E_0'} qE(E_0' - E)e^{-\mu_1'(E)M_1-\mu_2'(E)M_2}\mathrm{d}E, \end{cases} \tag{1}$$

where k is the transmission factor of the detector; E_0 and E_0' are maximum photon energies that equal to the low and high voltages (V_L and V_H), respectively; $\mu_1(E)$, $\mu_1'(E)$, $\mu_2(E)$ and $\mu_2'(E)$ are mass attenuation coefficients of the two components at V_L and V_H, respectively; and q is the charge activated by unit photon energy. The components of $k\int_0^{E_0} qE(E_0 - E)\mathrm{d}E$ and $k\int_0^{E_0'} qE(E_0' - E)\mathrm{d}E$ are the original signals before X-rays attenuation in the sample, and 100% detection efficiency is assumed. Obtaining the solutions of Eq. (1) means obtaining expressions for M_1 and M_2 as certain function of i and i'. It is a complex computing process when a huge of data are treated. Using the equivalent energy method, the coefficients of μ_1, μ_1', μ_2 and μ_2' become constants and Eq. (1) can be written as:

$$\begin{cases} \ln(i_0/i) = \mu_1(E_\mathrm{eq})M_1 + \mu_2(E_\mathrm{eq})M_2, \\ \ln(i_0'/i') = \mu_1'(E_\mathrm{eq}')M_1 + \mu_2'(E_\mathrm{eq}')M_2, \end{cases} \tag{2}$$

where $i_0 = k\int_0^{E_0} qE(E_0 - E)e^{-\mu_0(E)M_0}\mathrm{d}E$ and $i_0' = k\int_0^{E_0'} qE(E_0' - E)e^{-\mu_0'(E)M_0}\mathrm{d}E$ are the initial intensities of

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X-rays at V_L and V_H, respectively; i and i' are the intensities of X-rays transmission at V_L and V_H, respectively; $\mu_0(E)$ and $\mu_0'(E)$ are the mass attenuation coefficients of the filter at V_L and V_H, respectively; M_0 is the mass thickness of the filter; $\mu_1(E_\mathrm{eq})$, $\mu_1'(E_\mathrm{eq}')$, $\mu_2(E_\mathrm{eq})$ and $\mu_2'(E_\mathrm{eq}')$ are the equivalent mass attenuation coefficients of the two materials at the equivalent energies E_eq and E_eq', respectively. The equivalent coefficients are determined by factors of the voltages, filter material and sample thicknesses. Eq. (2) is linear in two unknown materials (M_1 and M_2) if the other parameters are identified. This reduces computing time and increases the computation accuracy. In this paper, the dual-component samples are comprised of different pieces of 0.35-mm thick aluminum and 2-mm thick plexiglass.

C. Monte Carlo Simulation

Monte Carlo simulation using EGSnrc packages are widely used in absorption simulation [6, 7]. It can simulate the transport of electrons and photons in an arbitrary geometry for particles from a few keV to several hundreds of GeV in energy [8]. The geometry configuration used in our simulation is shown in Fig. 1. A total of 10^8 particle tracks are simulated. The photons passing through samples are scored excluding the scattering ones.

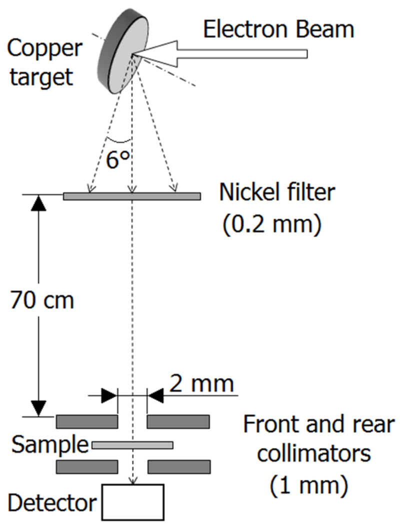

Fig. 1. Geometry configuration of the experiment.

D. Experiments

The experimental setup is shown in Fig. 1. The X-ray generator with a Cu target is from Dandong High-voltage device factory, Liaoning, China. A Ni filter of 0.2 mm thickness is used at low tube voltages, while 1-cm thick Cu is used at high tube voltages. Collimated X-rays pass through the dual-component sample of 0.35-mm aluminum and 2-mm plexiglass. The detector is of GOS scintillator and silicon photodiode. An HPGe detector with a 0.127-cm thick Al window is used, too.

E. Error analysis

In this section we will discuss the errors of the equivalent energy method at different tube voltages. From Eq. (2), M_1 and M_2 depend closely on mass attenuation coefficients of the two components. From NIST [9], both mass attenuation coefficients of aluminum and plexiglass decrease with increasing photon energy in 10–100 keV, but the coefficients are insensitive to the photon energy above 100 keV. So, low energy X-rays (E <100 keV) relatively play an important role on the coefficients. In this paper, high energy X-rays generated with tube voltage of 140 kV is preseted, and low tube voltages 50 kV, 60 kV, 70 kV, 80 kV and 90 kV are chosen to study their effects on the mass attenuation coefficients. Here, we will take the solution of M_1 as an example. According to Eq. (2), the solution of M_1 can be written as:

$$M_1 = [\mu_2 \ln(i_0'/i') - \mu_2' \ln(i_0/i)]/(\mu_1'\mu_2 - \mu_1\mu_2'). \quad (3)$$

Suppose the detector readings at different voltages are not correlated. For a tiny change in M_1, Eq. (3) can be written as:

$$\sigma_{M_1} = \{[(\sigma_{i'}/i')\mu_2]^2 + [(\sigma_i/i)\mu_2']^2\}^{1/2}/|\mu_1'\mu_2 - \mu_1\mu_2'|, \quad (4)$$

where $\sigma_{i'}/i'$, σ_i/i are the uncertainties of dose at V_H and V_L, respectively. They can be expressed as $1/i^{1/2}$, where i is the dose that is the integral of continuous spectrum over the entire energy domain. We note that only the uncertainties of the detector concerned as the effective dual-energy has been identified.

III. RESULTS AND DISCUSSION

A. Numerical calculations and M-C simulation

Figure 2 shows $\ln(i_0/i)$ as a function of mass thickness of aluminum and plexiglass, at tube voltage of 70 kV using 0.2-mm Ni filter, and at 140 kV using 1-cm Cu filter. The results were obtained by M-C simulation with the EGSnrc codes. The equivalent attenuation coefficients are similar in mass thickness range under investigation for the two materials, hence the feasibility of the equivalent energy method in mass thickness measurements for dual-component samples.

According to Eq. (2), μ_1, μ_1', μ_2 and μ_2' should be identified first to obtain the solutions of M_1 and M_2. Approximately, the equivalent mass attenuation coefficients of the two single-components can be used. Surely, this brings extra errors in the computations as X-rays are hardened more in the dual-component samples than in the single-component sample. The errors are studied by numerical calculations, at tube voltage of 70 and 140 kV, with different thickness combinations of the two components. The results are given in Table 1. The relative errors are less than 2% for two components when the X-ray energy is increased.

TABLE 1. Relative errors of mass thickness (g/cm^2) between specified values and numerical calculations

	Samples									
	Aluminum					PMMA				
Specified	0.473	0.378	0.284	0.189	0.095	0.236	0.472	0.708	0.944	1.180
Numerical	0.466	0.375	0.284	0.190	0.096	0.238	0.471	0.706	0.942	1.180
Relative errors (%)	1.48	0.79	0.00	0.53	0.05	0.85	0.21	0.28	0.21	0.00

TABLE 2. The detected and nominal mass thicknesses (g/cm^2) of Al and PMMA samples. The nominal values are provided by the manufacture

	Samples									
	Aluminum					PMMA				
Nominal	0.756	0.662	0.567	0.473	0.378	0.694	0.930	1.173	1.411	1.647
Detected	0.748	0.657	0.546	0.462	0.367	0.665	0.883	1.213	1.435	1.647
Relative errors (%)	1.11	0.82	3.70	2.33	2.96	4.23	5.08	3.39	1.70	0.00

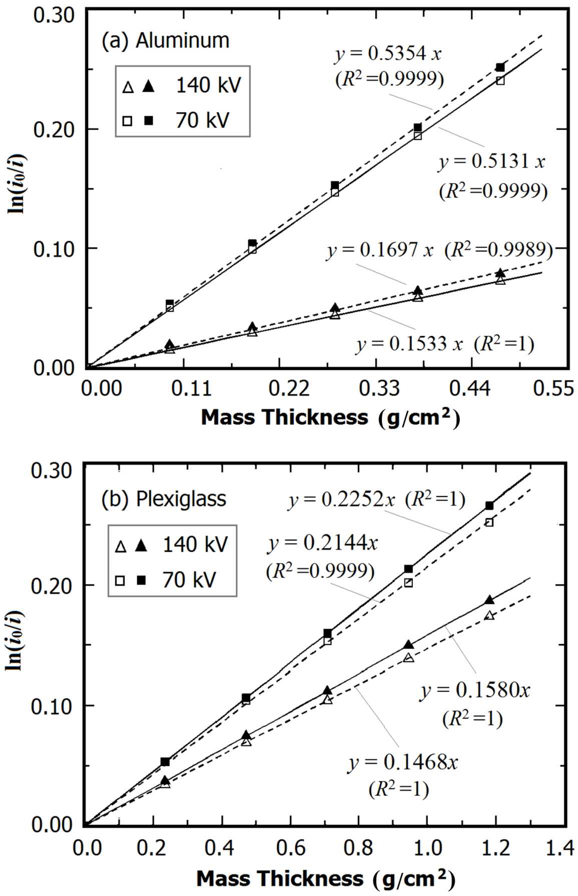

Fig. 2. Relationships between $\ln(i_0/i)$ and mass thickness of aluminum (a) and plexiglass (b) obtained by EGSnrc and numerical calculations.

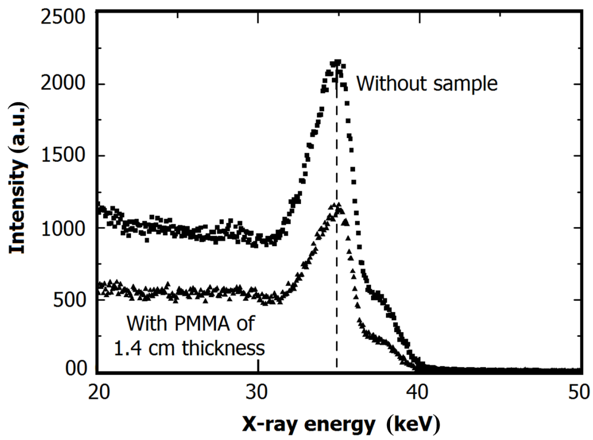

Fig. 3. X-ray spectra with tube voltage 35 kV measured by HPGe detector.

B. Experimental errors

The experimental results of mass thickness measurements for the dual-component samples (Al and PMMA) are given in Table 2. The two X-ray energies were 30 and 45 kV. The tube current was 30 mA to ensure enough intensity of X-rays. A 0.2-mm Ni filter was used. The detector output of over 10 000 was ensured to reduce the background. From Table 2, the errors are less than 5% in the thickness ranges of the dual-component under investigation.

For quantitative description of the degree of spectrum-hardening, X-ray spectra with and without a 1.4-cm thick PMMA sample were measured at 35 kV using an HPGe detector (Fig. 3). Peak positions of the two spectra are close to each other. So the equivalent energy of X-ray can be treated as a constant in the experiment using the HPGe detector with a 0.127-cm Al window. The results show that the equivalent energy in the experiments is near the peak position.

C. Optimal low voltage

The squared statistic counting error, $1/N$ (N is the number of photons), as a function of low voltages is shown in Fig. 4. The results were obtained by EGSnrc M-C simulation of different thickness combinations of Al and PMMA, assuming a 100% detection efficiency. The statistical errors are insensitive to the Al and PMMA thickness combinations

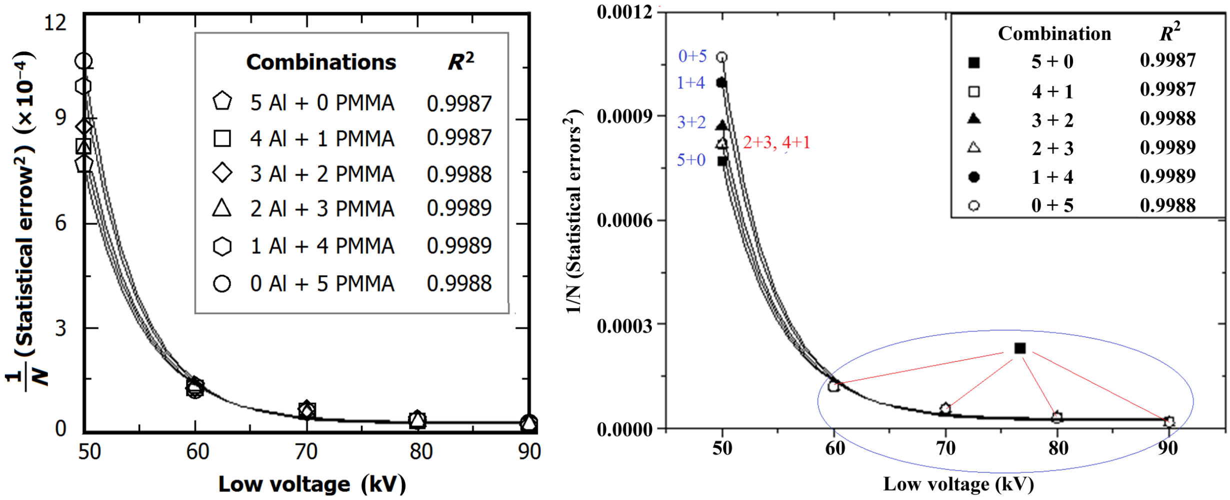

Fig. 4. Statistical errors in different thickness combinations of two materials at different low voltages.

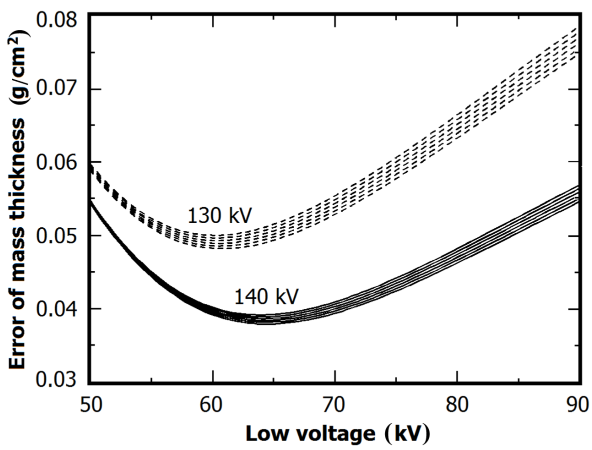

Fig. 5. Errors caused by different kV combinations with different thickness compositions of two components by numerical calculations.

in the thickness range under investigation. The curves of "2Al + 3PMMA" and "4Al + 1PMMA" overlap in Fig. 4. At low voltage of over 60 kV, the statistical errors of all the Al and PMMA thickness combinations are approximately constant.

The equivalent mass attenuation coefficients of Al and PMMA at low voltages of 50–90 kV are calculated, with 0.2-mm thick Ni filter. The results can be fitted by exponential functions of the low voltage (x) as ($R^2 > 0.999$):

$$\mu_1 = 4.36341e^{-x/23.21801} + 0.29753, \tag{5}$$

$$\mu_2 = 0.46335e^{-x/27.13392} + 0.18987. \tag{6}$$

Now, we can take the factors μ_1, μ_1', μ_2, μ_2', σ_i/i and $\sigma_{i'}/i'$ into Eq. (4) to calculate the errors of M_1 caused by the statistical errors. At high voltages of 130 and 140 kV, the results are shown in Fig. 5, where each curve corresponds to a thickness combination of Al and PMMA in Table 1. It can be seen that minimum error occurs at low voltage of about 60 and 65 kV for high voltage of 130 and 140 kV, respectively. Also, at high voltage of 130 kV, different choices of the low voltages will bring a wider range of error from 0.05 g/cm^2 to 0.08 g/cm^2.

We note that this paper involves just aluminum and plexiglass dual-component samples, as we are interested in human bone density measurements. Human body can be treated as dual-component samples similar to aluminum (bone mineral) and plexiglass (water or soft issue). However, the method shall be effective in other dual-component samples of different materials, if the equivalent energy can be found. And the numerical calculations model will be helpful when the materials are known.

IV. CONLUSION

Equivalent energy method is introduced in mass thickness measurements for dual-component samples. In this method, linear equations, rather than integral equations, are solved, hence a simple and fast approach, with low demand on detectors. Based on known composition of the dual-components, it can be applied to DEXA (dual-energy X-ray absorptiometry) and Dual-energy CT in diagnostics of either a human body or a printed circuit board. With knowledge of the nature of materials, feasibility of the equivalent energy method can be predicted by numerical calculations and give guidance in the experiments. The results of Monte Carlo simulation agree well with those by numerical calculations in this paper. In the range of Al and PMMA thickness combinations under investigation, relative errors of numerical results are less than 2%. Considering the counting statistical errors, an optimal low voltage is found for a preseted high voltage. As an effective way to examine mass thicknesses of dual-component samples, the equivalent energy method can be well applied in clinics and industries.

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